SHORT COMMUNICATION

REPLY: PEAT MOUNDS WITH NON-UNIFORM PROPERTIES

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ABSTRACT

In estimating the form of a peat mound from the equilibrium water table position, the assumption that hydraulic conductivity shows an exponential decline with depth (as used in the Armstrong (*Earth Surface Processes and Landforms*, 1995, **20**, 473–477) analysis) may be questioned in some circumstances. However, this does not invalidate the use of the hydrologic model for peat mounds, and results are presented which could be used to evaluate other forms of the depth-dependent relationship.

KEY WORDS hydraulic conductivity; Girinsky seepage potential

I thank Baird and Gaffney for their interest in my note (Armstrong, 1995). I accept their correction of my misuse of the relationship between hydraulic conductivity and methane release. This comment was added at a late stage when I saw a pre-publication copy of Baird (1995), but before Baird and Gaffney (1995) was published. I apologise to readers of *Earth Surface Processes and Landforms* for this error.

However, this does not in any way alter the main thrust of the note, which was to demonstrate that it is entirely possible to derive a hydrological model capable of predicting the form of the water table (and hence perhaps of the peat mound itself) in non-uniform material. In order to illustrate this point, I used the inverse exponential relationship for hydraulic conductivity. Baird and Gaffney argue that the field evidence for this relationship is not universally convincing, which I accept, although it certainly seems to be useful for some situations. My purpose in using the inverse exponential relationship was to present a reasonable relationship that could be used for model development. It was never my intention to claim general validity. Indeed it is to be expected in a medium as varied as peat, with complex depositional histories, that the hydraulic conductivity will vary with depth in many and varied ways. What is remarkable about the inverse exponential relationship is not that it fails to represent all circumstances, but just how many sets of data it does approximate. Despite the problems identified in reviewing the field data, there were few observational data sets that suggested an increase in hydraulic conductivity with depth, which would be required if the methane release mechanism were to be invoked.

The point of my note was, therefore, not to defend any specific relationship, but to demonstrate that it is possible to move away from simple uniform models of hydraulic conductivity to more realistic representations. Youngs' (1965) analysis, based on the Girinsky seepage potential, can be extended to many forms of relationship between hydraulic conductivity and depth, and indeed, if numerical techniques are used, to virtually any function. To recap, the flux through ditches spaced $2D$ apart in soils in which the hydraulic conductivity, $K(z)$, varies continuously as a function of height, z, above some datum, is given by the inequality:

$$\int_{H_w}^{H_m}\left[\int_0^H K(z)\mathrm{d}z\right]\mathrm{d}H > \frac{qD^2}{2} > \int_{H_w}^{H_m}\left[\int_0^H K(z)\mathrm{d}z\right]\mathrm{d}H - \int_0^{H_m}\left[\int_z^{H_m}\frac{q}{K(z)}\mathrm{d}z\right]K(z)\mathrm{d}z$$

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where H_w is the height of water in the ditches, and H_m is the maximum water table height, at mid-drain spacing (Youngs, 1965). I arrived at the results in my note by use of the left-hand side of this inequality, and noting the comment by Youngs (1965) that where the hydraulic conductivity increases strongly with height, then the second term in the right-hand side of the inequality is sufficiently small to be ignored. Youngs (1965) also presents solutions for other forms of the relationship $K(z)$, and these can be used in similar ways as the exponential function to describe the water table in non-uniform deposits.

Two points need, however, to be recapitulated. Firstly, Youngs presents an inequality which defines upper and lower bounds for the water table shape. For other forms of the $K(z)$ relationship, uses should identify the uncertainty involved in using either the left- or right-hand side of the inequality. Secondly, my analysis used a local definition of the relationship $K(z)$, rather than a global definition relevant to the whole peat mound, hence the requirement to use numerical techniques to solve for successive increments of distance away from the peat edge.

If appropriate, the hydrology of any situation can be approximately modelled by simply using the left-hand side of the inequality as if it were an equality:

$$q = \frac{2}{D^2} \int_{H_w}^{H_m} \left[\int_0^H K(z)\mathrm{d}z \right] \mathrm{d}H$$

The previous note used numerical methods to define the values of H_m that satisfied this relationship for specified values of q and H_w. The mean annual flux, q, is defined by the position within the mire (being calculated from the mean annual excess rainfall and the surface area), and the water table height at the outlet distance (H_w) is defined as the boundary conditions at the edge of the mire, and then redefined by the previously calculated position, as the calculation is repeated from the edge to the centre of the mound. This method can be extended to any form for the relationship $K(z)$ provided the user can integrate the function, either analytically or numerically.

For cases where the relationship is exponential ($K(z) = K_0 e^{\beta z}$) this gives us the relationship used in the previous note,

$$q = 2K_0 \left[(e^{\beta Hm} - 1)/\beta L - \left(\frac{H_m}{L} \right) \right] \Big/ \beta L$$

Other relationships are, however, possible. Youngs (1965) gives solutions for a number of other cases, but all for situations where the water level in the ditch, H_w, is zero. Thus for uniform soils ($K(z) = K_0$) we get the well known result:

$$q = K \frac{(H_m^2 - H_w^2)}{D^2})$$

equivalent to the original analysis of Ingram (1982). For linearly declining conductivity, $K(z) = K_0(z/D)$, we get:

$$q = K_0(H_m^3 - H_w^3)/3D^3$$

and for soils with hydraulic conductivity increasing with depth, $K(z) = K_0(1 + \alpha z)$, we get:

$$q = K_0(H_m^2 - H_w^2)/D^2 + \alpha(H_m^3 - H_w^3)/3D^3$$

Each of these could easily be inserted into the same analysis for water table form, and solved numerically.

It should, however, be noted that the scheme presented in my note uses a local definition of the $K(z)$ relationship. This means that the layers in the peat are parallel to the surface, and not parallel to the base. I suggest that is probably a more realistic assumption.

What Baird and Gaffney have done is to re-emphasize the need for field data describing the internal structure of peat mires, and in particular the variation of hydraulic conductivity with both depth and horizontal position. This can easily be measured using a technique such as the piezometer method (Bouwer and Jackson, 1974).

REFERENCES

Armstrong, A. C. 1995. 'Hydrological model of peat-mound form with vertically varying hydraulic conductivity', *Earth Surface Processes and Landforms*, **20**, 473–477.

Baird, A. J. 1995. 'Hydrological investigations of soil water and groundwater processes in wetlands', in Hughes, J. M. R. and Heathwaite, A. L. (Eds), *Hydrology and Hydrochemistry of British Wetlands*, Wiley, Chichester, 111–129.

Baird, A. J. and Gaffney, S. W. 1995. 'A partial explanation of the dependence of hydraulic conductivity on positive pore water pressure in peat soils', *Earth Surface Processes and Landforms*, **20**, 561–566.

Bouwer, H. and Jackson, R. D. 1974, 'Determining soil properties', in van Schilfgaarde, J. (Ed.), *Drainage for Agriculture*, American Society of Agronomy, Madison, Wisconsin, 611–672.

Ingram, H. A. P. 1982, 'Size and shape in raised mire ecosystems: geophysical model', *Nature*, **297**, 300–303.

Youngs, E. G. 1965. 'Horizontal seepage through unconfined aquifers with hydraulic conductivity varying with depth', *Journal of Hydrology*, **3**, 283–296.